Ambiguity, Asymmetric Information and Irreversible investment

Yao Wang[a], Hai Zhang [b] and Shunming Zhang[c]

[a] *School of Finance, Renmin University of China, Beijing, China;*
[b] *Department of Accounting & Finance, Strathclyde Business School, Glasgow, UK;*
[c] *China Financial Policy Research Center and School of Finance, Renmin University of China, Beijing, China*

Abstract

We develop a signal game model of investment to examine the implications of ambiguity aversion on corporate equilibrium strategies, investment dynamics, and financing decisions within incomplete markets marked by asymmetric information. Our analysis reveals that firms with higher risk profiles exhibit a preference for less efficient separating or pooling equilibrium strategies, leading to heightened adverse selection costs, increased financing costs, and distorted investment decisions. Notably, ambiguity aversion exerts a comparable yet more pronounced influence, magnifying financing costs, adverse selection costs, and introducing distortion in investment choices. This heightened ambiguity aversion escalates the likelihood of inefficient separating and pooling equilibria, ultimately resulting in a discernible welfare loss. The findings underscore the substantial impact of ambiguity aversion on strategic decision-making and equilibrium outcomes in the context of investment within environments characterized by information asymmetry and incomplete markets.

Keywords: Ambiguity aversion, Asymmetric information, Equity-for-guarantee swap, Signal game, Irreversible investment
JEL: G11, G14, G32

1. Introduction

Ambiguity, also recognized as Knightian uncertainty, is progressively assuming a crucial role in economic and financial modelling. Unlike risk, ambiguity lacks a calculable or observable probability distribution, and an economic agent cannot determine the precise probability measure of a set of economic data. Since the seminal contributions of Knight (1921) and Ellsberg (1961), the literature analysng models that incorporate ambiguity parametrically has substantially evolved. A widely cited result in this literature suggests that, in the presence of ambiguity, future market conditions become uncertain, rendering it impossible for the firm's decision-makers to accurately estimate project cash flows when making investment and financing decisions.

This uncertainty leads to an inconsistency of interests between decision-makers (management) and owners (shareholders), necessitating decision-makers to account for the ambiguous aversion attitude of shareholders. Building upon the multiplier preferences introduced by

Hansen and Sargent (2001), this paper formulates a robust model applicable to continuous time frames, capturing decision-makers' concerns about ambiguity or model uncertainty. This induces decision-makers to make robust decisions against potentially misspecified models. The paper's novelty lies in studying how shareholders' preference for robustness influences corporate investment decisions and valuation.

A substantial body of literature underscores the significance of incorporating time-varying volatility into various macroeconomic time series. The dynamic evolution of volatility is highly intricate, and existing volatility models exhibit notable shortcomings in accurately capturing the underlying parameter motion laws. This raises concerns about the models' ability to precisely represent these motion laws. In this study, we delve into the inherent impact of volatility ambiguity, grounded in the cash flows of investment projects, on investment behaviour and valuation. The cash flow post-investment adheres to arithmetic Brownian motion, ensuring the validity of our findings even when dealing with negative cash flows, as is often the case for start-ups and small to medium-sized enterprises (SMEs). Inspired by efforts to alleviate financing constraints, we introduce a novel financing instrument, equity-for-guarantee swaps (EGS), which has been utilized in the Chinese credit guarantee market since 2002 and surpasses traditional credit guarantee schemes in markets with perfect information (see, e.g., Yang and Zhang, 2013; Wang, Yang and Zhang, 2015; Song, Zhang and Zhao, 2021; Wang et al., 2023).

Information plays a pivotal role in the seamless functioning of financial activities. Ambiguity arises precisely from the decision maker's lack of adequate information. For external investors, the ability to perfectly, accurately, and timely grasp financiers' information largely determines the feasibility of transactions, cost considerations, and overall efficiency improvements. Almost all financial transactions grapple with varying degrees of information asymmetry, giving rise to moral hazard and adverse selection—two significant challenges in contemporary financial practice. Some current challenges faced by the financial system, such as the difficulties and high costs of financing for private enterprises and small and micro-enterprises, can be traced back to the impact of information asymmetry.

Addressing the information asymmetry dilemma in financial markets and understanding how ambiguity influences a firm's investment decisions in the absence of perfect observation of the firm's growth potential are critical questions. The literature on optimal security design under asymmetric information dates back to Myers and Majluf (1984). They posit that information asymmetry in the capital market is a key factor driving managers to opt for the abandonment of investment projects with positive net present value (non-participation). Our analysis introduces an alternative perspective, suggesting that firms' non-participation in investment stems from increased ambiguity associated with the volatility of investment projects. This ambiguity diminishes the benefits of investment options and raises financing costs. Employing signal game theory, we quantitatively analyse and assert that, to mitigate information asymmetry, ambiguity intensifies the distortion in investment decisions. More specifically, we find that higher risk firms tend to choose a less efficient separating or pooling equilibrium strategy characterized by elevated adverse selection costs, a surge in financing costs, and investment distortion. Similarly, the introduction of ambiguity aversion has a notable impact by amplifying financing costs, adverse selection costs, and inducing distortion in investment decisions. This escalation in ambiguity aversion heightens the probability of inefficient separating and pooling equilibria, ultimately resulting in welfare loss. Significantly, the effect of ambiguity aversion is even stronger than that of firm risks. This is due to the fact that a high level of ambiguity aversion diminishes the project value and amplifies the imitation benefits for low-type firms, prompting high-types to invest more effort in separation, which in turn leads to a smaller first-best equilibrium region.

The continuous-time version of multiple priors (or max-min expected) utility introduced by Gilboa and Schmeidler (1989) stands out as one of the most widely adopted and effective models for characterizing ambiguity. Building on this foundation, Hansen and Sargent (2001) establish connections between max-min expected utility theory and applications in robust control theory, elucidating multiplier preferences. Further extending the robust method into a continuous-time framework with a robust Hamilton-Jacobi-Bellman (HJB) equation, Anderson et al. (2003) contribute to this literature.

The decision-theoretic framework of this paper draws from Maccheroni et al. (2006), incorporating multiple priors preferences from Gilboa and Schmeidler (1989) and multiplier preferences from Hansen and Sargent (2001). They provide an axiomatization of variational preferences, asserting that variational preferences inherently exhibit ambiguity aversion. This offers a theoretical foundation for understanding the ambiguous aversion exhibited by firms.

In the realm of corporate finance theory, there is a growing interest in unravelling the impact of ambiguity on intricate decision-making processes within corporations, encompassing investment, financing, and compensation. Nishimura and Ozaki (2007) find that heightened risk and ambiguity elevate the value of waiting options, prompting companies to delay investments, albeit for distinct reasons. Following this, Thijssen (2011) applies the multiple priors model to construct a continuous-time model, suggesting that ambiguity, all else being equal, tends to delay investment. Unlike risk, an increase in ambiguity diminishes the value of investment opportunities, leading managers grappling with greater ambiguity to curtail capital expenditure and increase cash holdings (Neamtiu, Shroff, White and Williams, 2014; Agliardi, Agliardi and Spanjers, 2016; Luo and Tian, 2022). Ambiguity aversion not only reduces average investment but also influences optimal pay-performance sensitivity, as reflected in robust contracts between entrepreneurs and investors (Ling, Miao and Wang, 2021). Supporting ambiguity from both theoretical and empirical perspectives, Chen et al. (2023) contributes to explaining corporate debt puzzles and demonstrates that tradeoff models incorporating ambiguity aversion better align with real-world data.

While existing literature typically incorporates ambiguity into investment and financing decision models through a form of drift uncertainty, it's crucial to note that Epstein and Ji (2013) makes a crucial distinction between ambiguity about volatility and ambiguity about drift, emphasizing the necessity for more research on volatility ambiguity. This study contributes to the existing literature by delving into the endogenous effects of market-based ambiguity about volatility on corporate decision behaviour. Departing from existing models, our ambiguity model distinguishes between drift ambiguity and volatility ambiguity, capturing the amplifying effect of ambiguity on asymmetric information.

This paper builds upon a substantial body of literature exploring models of signal games under asymmetric information. Adopting a perspective similar to Flannery (1986), we assume that corporate insiders possess more information about the firm than outsider investors. High-quality firms can effectively convey true information to the market, albeit incurring positive transaction costs. In contrast to most existing literature, which identifies firms capable of raising all outside funds for investment projects, this paper emphasizes financing-constrained firms employing EGS as a financing tool. Consequently, the asymmetry in information shifts from insiders and investors to insiders and insurers, with adverse selection costs manifesting as equity dilution resulting from the distortion of guarantee costs. This distinguishes our work from existing literature, overlooking the study of ambiguous volatility.

This paper is organized as follows. In Section 2, we establish the corporate investment model specification incorporating ambiguity. delineates the solutions to the robust control problem for participants and elucidates the first-best equilibrium. The exploration of optimal investment strategies under asymmetric information is conducted in Section 4. Finally, Section 5 concludes.

2. Model and assumptions

This paper consider a young firm can implement an investment opportunity by paying a fixed sunk cost $I > 0$ at any time t. The firm raise the funds needed for the investment from outside, which leads to the problem of information asymmetry. Firm insiders know more about firm quality and project quality than potential investors, but management may not be able to accurately assess project benefits, that is, the firm is ambiguous about project cash flow. This leads to shareholders distrusting the pricing model provided by the management. Management make decisions based on maximizing shareholder value. Investors rationally analyse the decision-making behaviour of management, so as to judge the quality of firm and project.

2.1. Firms and projects

We focus on startups and young firms without a well-established credit history. These firms as borrowers are penniless, and each must borrow sunk cost I to invest. Time is continuous, infinite, and indexed by $t \geq 0$. After the investment is implemented, the project generates an observable continuous stream of cash flows x_t. We can interpret x_t as instantaneous cash flows that evolves according to:

$$\mathrm{d}x_t = \mu \mathrm{d}t + \sigma \mathrm{d}Z_t$$

where the drift rate μ and the volatility $\sigma > 0$ are constant over time. $(Z_t)_{t\geq 0}$ denotes a standard Brownian motion on the filtered probability space $(\Omega, \mathcal{F}, Q)$ with filtration $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions and the risk-neutral probability measure Q. Under measure Q, the financial market is dynamically complete and arbitrage-free and that there is a constant risk-free interest rate r.

The stochastic process, $X = (x_t)_{t\geq 0}$, captures shocks in the cash flow, assume that $x_0 = x$ is the initial value of cash flow. Denote by $\Pi(x)$ the present value of a perpetual stream of cash flows starting at time $t = 0$:

$$\Pi(x) = \mathbb{E}\left[\int_0^{\infty} e^{-rs} x_s \mathrm{d}s \,\middle|\, x_0 = x\right] = \frac{\mu + rx}{r^2} \tag{1}$$

Similarly, $F = \int_0^{\infty} e^{-rs} f \mathrm{d}s = f/r$ represents the present value of operating expenses, where $f > 0$ represents constant operating expenses. In this setting, at any time t after investment, a firm generates a profit flow given by $\Lambda x_t - f$, Λ is cash flow scaling factor that represents firm quality. The firm is of heterogeneous quality k, modelled as differences in the scaling of cash flow. Specifically, assume there are two types of firms in financial markets, a high-type firm ($k = h$) means a kind of firm that develops fast in a long period of time, can bring high benefits, has high value-added and high return on investment, whereas a low-type firm ($k = l$) is worse than high-type firms. Rewrite the profit flow as $\Lambda_k x_t - f$, where $\Lambda_k = \{\Lambda_l, \Lambda_h\}$ with $0 < \Lambda_l < \Lambda_h$. Under symmetric information, firm type is observable for both the firm owner and external investors. But under asymmetric information, the type of firm is private information that is known only to those inside the firm, outside investors use Bayes' rule to update their beliefs based on firm behaviour. The public information is the market's belief $p = \Pr(\Lambda_k = \Lambda_h) \in (0, 1)$. Hence, before investment, the market believes that the firm's quality is $\Lambda_p = p\Lambda_h + (1 - p)\Lambda_l$.

The firm has discretion over the timing of investment as well as the timing and type of security issuance. In order to alleviate financing constraints and increase equity value, firm turns to an insurer and signs a three party agreement with an investor and an insurer called Equity-for-guarantee swap (EGS) which avoid the strong assumption of Morellec and Schürhoff (2011) about firms have enough resources to fund investment. Under the EGS contract, in order to implement the investment smoothly, the enterprise applies for loan I from the investor. Considering the insufficient repayment ability of the enterprise, the guarantor is introduced, who can repay the part of the loan I that the enterprise cannot afford. The requirement of the guarantor is to obtain a certain percentage of the equity to protect its own interests. Figure 1 shows the corresponding contract relationship.

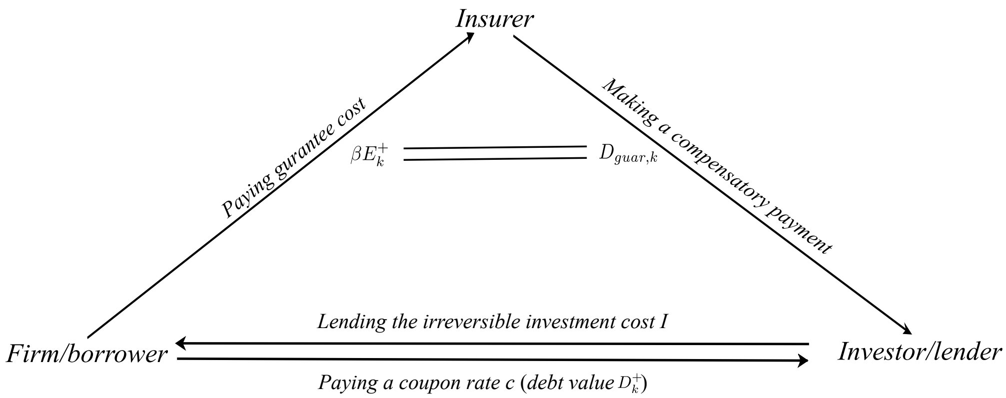


Figure 1: EGS contractual relationship. E_k^+ is the value of the equity after investment. $D_{guar,k}$ represents the values of the insurer's compensatory payment to the investor, that is, the loan undertaken by a insurer that the firm cannot repay. β stands for guarantee cost, representing the percentage of equity value received by the insurer.

2.2. The benchmark model specification with ambiguity

Different from the previous literature on real-options-based irreversible investment problem, in the settings of this article, shareholders believe that the cash flow of investment projects is a real-valued stochastic process generated by a Brownian motion, but the measure of Brownian motion is uncertain. This assumption means that the cash flow model under the risk-neutral probability measure Q provided by management is not trusted by shareholders. That is, there might exist misspecification or ambiguity for the model of the firm's cash flow shock. Then the firm's owner faces the so-called Knightian uncertainty, in which the owner is faced with a set of probability measures instead of a single probability measure. The shareholder may think that the cash flow is generated by alternative probability measures Q^θ which slightly different from Q.

To characterize model ambiguity, we follow Maccheroni et al. (2006) and propose real-options-based volatility ambiguity robust contract problem. Define a Radon-Nikodym derivative (m_t) of Q^θ with respect to Q. For any continuous function g in Ω,

$$m_t(g) = \left(\frac{\mathrm{d}Q^\theta}{\mathrm{d}Q}\right)(g).$$

This leads us to define a progressively measurable process $\theta = \{\theta_t\}_{t\geq 0}$ called density generator that satisfies:

$$\Pr\left\{\int_0^t |1/\theta_s|^2\,\mathrm{d}s < \infty\right\} = 1$$

for any $t > 0$. And Z^θ solves the differential equation $\mathrm{d}Z_t = \theta_t \mathrm{d}Z_t^\theta$ where $\mathrm{d}Z_t^\theta$ is a Brownian Motion under Q^θ. Intuitively, if the shareholder is ambiguity averse, then $\theta_t \geq 1$, and the deeper the shareholder's ambiguity aversive degree, the greater the θ_t. The set of probability measures which can be generated using $\Theta, Q^\Theta = \{Q^\theta : \theta \in \Theta\}$, is rectangular. By modelling alternative probability distributions, shareholders can consider alternative models to protect himself from model misspecifications. And it allows us to reformulate the stochastic process of shocks in the cash flow as

$$\mathrm{d}x_t = \mu \mathrm{d}t + \theta_t \sigma \mathrm{d}Z_t^{\theta}.$$

It describe the movement of cash flow x_t under perturbation probability measures, are different in the volatility term. Under ambiguity characterized by Θ, the shareholder considers all cash flow process, with $\theta \in \Theta$ varying. Affected by ambiguity averse about the measurement of cash flow, shareholder considers perturbations of Q and maximizes the minimum expected value under the perturbed measures of the project's value.

2.3. Relative entropy

Based on the distrust of the management, the shareholders distort the probability measure from Q to Q^{θ}, resulting in the misspecification of the model and the loss of the value of the shareholders. Therefore, it is necessary to set the entropy penalty function to compensate for the loss.

Generalizing the relative entropy cost of Anderson et al. (2003), Hansen et al. (2006), we specify the penalty term as

$$K_t(Q^{\theta}) = \mathbb{E}^{Q^{\theta}}[\int_t^{t(x^d)} e^{-r(s-t)} \Psi(h_s, x_s) \mathrm{d}s] \tag{2}$$

where $\Psi(\theta_t, x_t) = (1-\tau)\frac{\pi x_t}{2} \log\left(\frac{e^{\theta_t^2 - 1}}{\theta_t^2}\right)$ measures the size of the entropy cost in which τ is effective tax rate, $1-\tau = (1-\tau_c)(1-\tau_d), \tau_c$ is corporate profits tax rate, and τ_d is effective dividends tax rate. The π parameter represents the degree of ambiguity aversion of the firm. Specifically, as π increases, the degree of ambiguity aversion decreases. And when π approaches infinity, shareholders have no ambiguity, at which point Ψ equals 0 with $\theta_t = 1$. This assumption captures the intuitive idea that more ambiguity aversive shareholders should have a greater entropy penalty (measured by equity value). Similarly, the modelling of Ψ implies that the shareholders' aversion to cash flow uncertainty matters for the value of equity. The larger the value of π, the smaller the penalty term given in (2), and hence the more the shareholders trust the manager's probability model that also indicates the smaller the degree of firm's aversion to ambiguity.

3. Model solutions

Let us consider now the firm's investment decision problem under ambiguity, as outlined in the preceding subsection, employing dynamic programming. This entails addressing two pivotal decisions. The first involves determining the investment threshold – specifying when to initiate the investment project and activate the accompanying financing contract. The second decision revolves around establishing the default threshold – pinpointing when to discontinue firm operations post-investment, a juncture where shareholders are unable to safeguard the firm's robust growth, leading to investors assuming control of the residual firm value. To unravel these complex issues, we initiate the process by formulating a robust control problem involving the three key participants, debt holders (investors), insurers, and shareholders. Building upon this foundation, we proceed to derive the investment threshold and default threshold.

3.1. Investors' and insurers' problems

Investors' problems First, let's focus on investors and insurers. In the context of perfect information, EGS emerges as the optimal funding choice, compensating for the dead-weight loss stemming from the financing-constrained firm's inadequate funding capacity. EGS contracts enhance investor participation by offloading risk to the insurer. Since investors are assumed to be risk-neutral and all funds extended to the firm can be safeguarded, the ambiguity surrounding the project's cash flow does not impact the investors' interests.

During the duration of the project, the investor as the debt holder receive the continuous coupon payment c_k provided by the firm. If the firm defaults, the investor will take over the firm and get the post-default firm value. Then, based on the optimal trigger strategy made by the management with the goal of maximizing the interests of shareholders, the debt value can be given by the following formula.

$$\begin{aligned} D_k^+(x_t) &= \mathbb{E}^{Q^\theta}\left[\int_t^{t\left(x_k^d\right)} e^{-r(s-t)}(1-\tau_i)\,c_k\mathrm{d}s + \int_{t\left(x_k^d\right)}^{\infty} e^{-r\left(s-t\left(x_k^d\right)\right)}(1-\tau)\left[(1-\alpha)\,x_s - f\right]\mathrm{d}s\right] \\ &= (1-\tau_i)\frac{c_k}{r} - \left[(1-\tau_i)\frac{c_k}{r} - (1-\tau)\left[(1-\alpha)\Lambda_k\Pi\left(x_k^d\right) - F\right]\right]e^{-\xi_2\left(x_t - x_k^d\right)}, \end{aligned} \tag{3}$$

$\xi_{1,2} = \frac{\mu \mp \sqrt{\mu^2 + 2r\theta_t^2\sigma^2}}{\theta_t^2\sigma^2}$, where τ_i is personal tax rate, $t(x_k^d) = \{t : x_t \le x_k^d\}$ represents the time of default determined by management and x_k^d is the default threshold of type k firm. The firm immediately executes the default as soon as the cash flow down hits x_k^d. In addition, default is typically costly, so we assume that a fraction, α, of the cash flow related to x_t is lost.

Insurers' problems EGS, as a common type of atypical guarantee in transaction practice, is conducive to expanding the financing channels of Small and medium-sized enterprises and improving the ranking of "access to credit" in the World Bank's Global Doing Business Assessment. In terms of the protection of investors' interests, EGS guarantees the realization of investor's rights by requiring the insurer to compensate all the loss of interests at one time. So, the expected present value of guarantee under EGS contract, denoted by $I_k^+(x_t)$, is equal to the expected present value of equity that the firm-provided, minus the expected present value of the insurer's compensatory payment to the investor, where the compensatory payment is defined as sunk cost minus debt recovered value over the life of the project.

From the perspective of insurer's interests protection, the value of equity as the subject matter of the guarantee is volatile and depends on the stable operation of the investment project. Therefore, it is necessary for the insurer to act as a nominal shareholder to prevent the improper operation of the firm before providing compensation, which means that the insurer is subject to ambiguity same as shareholder. Assuming that the degree of ambiguity aversion of the insurer is identical to the shareholder, then taking the firm's optimal investment threshold x_k^i and default threshold x_k^d as given, the expected present value of the guarantee satisfies

$$I_k^+(x_t) = \beta_k(x_k^i)E_k^+(x_t) - \mathbf{1}_{\{x_t \le x_k^d\}} D_{guar,k}(x_t)$$

$$D_{guar,k}(x_t) = I - D_k^+(x_t)$$

Given that the profit flow accruing to debt holders after investment over each interval of time of length $\mathrm{d}t$ is given by c_k, the standard no-bubbles condition associated with the debt value is $\lim_{x\to\infty} D_k^+(x) = (1-\tau_i)\, c_k/r$. In the meantime, we define $I \equiv D_0(c_k) \equiv \frac{c_k}{r}(1-\tau_i)$, then $c = c_k = Ir/(1-\tau_i)$ and

$$D_{guar,k}(x_t) = (1-\tau_i)\left[\frac{c}{r} - (1-\tau)\left[(1-\alpha)\Lambda_k \Pi(x_k^d) - F\right]\right] e^{-\xi_2\left(x_t - x_k^d\right)}$$

. EGS should follow the principle of fairness and not harm the interests of the insurer, so the value of the certain fraction of equity that the insurer receives is equal to the insurer's compensatory payment at investment. Thus, the guarantee fee/cost is given by $\beta_k(x_k^i) = \frac{D_{guar,k}\left(x_k^i\right)}{E_k^+(x_k^i)}$.

3.2. Shareholders' problems

We establish the optimisation problem of irreversible investment under continuous time with ambiguity in which the objective of optimization is to maximize the value of equity. Let $E_k^+(x_t)$ be the equity value function after investment in the project. By a standard argument, the equity value from undertaking the project includes not only the entropy penalty cost $\Psi(\theta_t, x_t)$ caused by the shareholders' concerns for probability measure misspecification, but also the project profit flow $(1-\tau)(\Lambda_k x_t - f - c_k)$ in which c_k is coupon payment paid to the investor over the life of the project.

Building on the robust control models of Anderson et al. (2003) and Maccheroni et al. (2006), we represent the present values of equity after investment at time t as follows:

$$E_k^+(x_t) = \sup_{t(x_k^d)} \inf_{\theta_t} \mathbb{E}^{Q^\theta}\left[\int_t^{t(x_k^d)} e^{-r(s-t)}(1-\tau)(\Lambda_k x_s - f - c_k)\mathrm{d}s\right] + K_t(Q^\theta) \tag{4}$$

Decision makers want their decisions to maximise the minimum sum of the present value of expected payoffs under Q^θ and the entropy penalty term $K_t(Q^\theta)$. It is known that Q^θ is taken from Q^Θ and the perturbation of Q^θ represents a change in the degree of ambiguity aversion. According to the analysis, the first term of the present value of equity is smaller and the second term is larger as the degree of ambiguity aversion increases. Then the tradeoff requires the decision to be robust to the "worst-case" model in Q^Θ. $K_t(Q^\theta)$ is given by (2). Rewrite (4) as

$$E_k^+(x_t)=\sup_{t(x_k^d)}\inf_{\theta_t}\mathbb{E}^{Q^\theta}\left[\int_0^{t(x_d)} e^{-rt}(1-\tau)(\Lambda_k x_t - f - c)\mathrm{d}t + \int_0^{t(x_d)} e^{-rt}(1-\tau)\frac{\pi x_t}{2}\log\left(\frac{e^{\theta_t^2-1}}{\theta_t^2}\right)\mathrm{d}t\right]$$

Applying Itô's lemma, it is straightforward to demonstrate that $E_k^+(x_t)$ satisfies the following ODE before default,

$$rE_k^+(x_t) = \sup_{t(x_k^d)}\inf_{\theta_t} \mu\frac{\partial E_k^+}{\partial x_t} + \frac{\sigma^2\theta_t^2}{2}\frac{\partial^2 E_k^+}{\partial x_t^2} + (1-\tau)\left\{\left[\Lambda_k + \frac{\pi}{2}\log\left(\frac{e^{\theta_t^2-1}}{\theta_t^2}\right)\right]x_t - f - c\right\}. \quad (5)$$

Now, we focus on the solution of ODE (5) and summarize it into following proposition which characterizes the optimal robust decision.

Proposition 1. **(*First−best equilibrium*)** *Suppose that $E_k^+(x_t) > 0$ satisfies the ordinary differential equation (5) subject to the following boundary conditions:*

Condition 1. (value-matching condition) $E_k^+(x_k^d) = 0$
Condition 2. (smooth-pasting condition) $\left.\frac{\partial E_k^+}{\partial x_t}\right|_{x_t=x_k^d} = 0$
Condition 3. (no-bubble condition) $\lim\limits_{x_t\to\infty} E_k^+(x_t) < \infty$
Condition 4. (smooth-pasting condition) $\left.\frac{\partial V_k^+}{\partial x}\right|_{x_t=x_k^i} = \left.\frac{\partial V_k^-}{\partial x}\right|_{x_t=x_k^i}$*, where $V_k^+(x)$ is the value of the firm after investment given by $V_k^+(x_t) = E_k^+(x_t) + D_k^+(x_t)$, V_k^- is the value of the firm before investment given by $V_k^-(x_t) = (E_k^+(x_k^i) + D_k^+(x_k^i) - I)e^{-\xi_1\left(x_t - x_k^i\right)} = (1-\beta_k(x_k^i))E_k^+(x_k^i)e^{-\xi_1\left(x_t-x_k^i\right)}$ respectively.*

Then:

(i) The equity value function is $E_k^+(x_t)$. The worst-case density generator is given by

$$\theta_k^*(x_t) = \left(\frac{(1-\tau)\pi x_t}{(1-\tau)\pi x_t + \sigma^2\frac{\partial^2 E_k^+}{\partial x_t^2}}\right)^{\frac{1}{2}} \quad (6)$$

The optimal investment threshold satisfies

$$E_k'(x_k^i) + E_k^+(x_k^i)\xi_1 = (\xi_1-\xi_2)\left[(1-\tau_i)\frac{c}{r} - (1-\tau)\left[(1-\alpha)\Lambda_k\Pi\left(x_k^d\right) - F\right]\right]e^{-\xi_2\left(x_k^i - x_k^d\right)},$$

and the optimal default threshold satisfies

$$E_k'(x_k^d) = E_k^+(x_k^d) = 0$$

(ii) The project's cash flow x_t satisfies the following diffusion process:

$$\mathrm{d}x_t = \mu \mathrm{d}t + \theta_k^* \sigma \mathrm{d}Z_t^{\theta^*}.$$

The optimal entropy cost is

$$\Psi_k^*(\theta_k^*(x_t), x_t) = (1-\tau)\frac{\pi x_t}{2} \log\left(\frac{e^{\theta_k^{*2}(x_t)-1}}{\theta_k^{*2}(x_t)}\right).$$

(iii) If $\pi \to \infty$, then the solution of the problem is equivalent to the non-ambiguity solution.

The investment decision hinges on maximizing shareholders' interests, a process that entails addressing the decision maker's rational anticipation of potential concerns related to probability measure misspecification and project abandonment by shareholders. This introduces a challenging scenario wherein we must simultaneously consider equity value, default threshold, investment threshold, and entropy cost. As a result, numerical methods are imperative for resolving this intricate problem.

3.3. Discussion

Proposition 1 shows that there exists an optimal robust contract. Then, we discuss the properties and intuition of the result given by Proposition 1. We use the following parameter values: the risk-free rate $r = 5\%$, the volatility and growth rate of cash flow shock: $\sigma = 0.25$ and $\mu = 0.1$, operating leverage $F = 10/r$, the growth potential of the k type firms $\Lambda_k = 2$, sunk cost $I = 20$, default cost $\alpha = 0.5$, tax rates: $\tau_i = 0.35$, $\tau_c = 0.35$ and $\tau_d = 0.2$, respectively.

Equity value T The firm's management is dedicated to operating with a responsible attitude towards all shareholders, aiming to enhance overall shareholder value. In light of shareholders' concerns regarding the potential misspecification of probability measures, management decisions, such as those outlined in Proposition 1, have been made. Condition 4 provides the firm's value before investment, equivalent to the present value of equity before investment.

Figure 2 illustrates the equity value as a function of the volatility of the cash flow shock (σ), the growth potential of type k (Λ_k), and the growth rate (μ). The solid red line represents the equity value of non-ambiguous firms ($\pi \to \infty$), the blue dotted line corresponds to a firm with a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line depicts the equity value of a firm with a high degree of ambiguity aversion ($\pi = 1$).

Figure 2 unveils the following insights. The uncertainty surrounding cash flow generated by investment projects diminishes the equity's value, and this decrease is accentuated with heightened ambiguity aversion. In the figure, this trend is depicted by a solid red line at the top, a dashed blue line in the middle, and a dotted blue line at the bottom. Figure 2(a) indicates that an escalation in ambiguity leads to a reduction in equity value, while an increase in volatility corresponds to an augmented equity value. This aligns with the findings of Nishimura and Ozaki (2007), highlighting that the impact of ambiguity on the value of irreversible investment opportunities differs significantly from the impact of traditional risk. Figure 2(b) illustrates the interplay between ambiguity and the firm's growth potential on equity value. It demonstrates that when the degree of ambiguity aversion is low, the influence of ambiguity on equity value gradually diminishes with the improvement of firm quality.

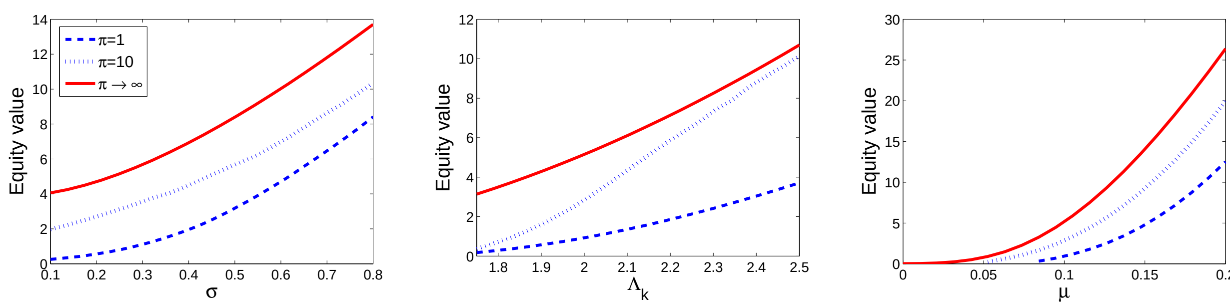
Interestingly, the presence of ambiguity may result in non-participation in the firm's investment projects. Firms might forgo valuable investment opportunities because the implementation of projects can generate a negative equity value, as shown in Figure 2(c). These findings can be formalized in the following proposition.

Proposition 2. *For type k firms, $k = h, l$,*

1. The equity value experiences a decline (rise) with an increase in ambiguity aversion (π): $\frac{\partial E_k^+}{\partial \pi} > 0$, while the equity value increases with the volatility of the cash flow shock σ: $\frac{\partial E_k^+}{\partial \sigma} > 0$.

2. A higher cash flow scaling factor Λ_k results in higher equity value, especially for firms with low ambiguity aversion (high π), i.e., $\frac{\partial E_k^+}{\partial \Lambda_k \partial \pi} > 0$.

3. The higher the growth rate μ, the higher the equity value: $\frac{\partial E_k^+}{\partial \mu} > 0$. Particularly, for firms with ambiguity aversion, if μ is sufficiently small, a non-participating investment region emerges where the equity value is less than or equal to 0.



(a) Volatility (b) Cash flow scaling (c) Growth rate

Figure 2: Equity Value. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on equity value under the first-best equilibrium scenario (symmetric information). The solid red line depicts the equity value of non-ambiguous firms ($\pi \to \infty$), the blue dotted line represents that of a firm with a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line showcases that of a firm with a high degree of ambiguity aversion ($\pi = 1$). The base parametrization is $r = 0.05$, $\sigma = 0.25$, $\mu = 0.1$, $F = 10/r$, $\Lambda_k = 2$, $I = 20$, $\alpha = 0.5$, $\tau_i = 0.35$, $\tau_c = 0.35$, $\tau_d = 0.2$.

The worst-case density generator, denoted as θ_t, serves as a density generator allowing the identification of any measure Q^θ based on its density generator θ_t. An increase in θ_t

signifies a more severe concern among shareholders regarding the misspecification of probability measures, negatively impacting the project. Simultaneously, an elevated θ_t contributes to higher entropy costs for shareholders, ultimately benefiting them. The worst-case density generator is a crucial component in solving equation (6), representing the pinnacle of model misspecification.

Figure 3 illustrates that the worst-case density generator surpasses 1, aligning with the hypothesis of ambiguity aversion. This implies that under ambiguity aversion, volatility consistently exceeds the true volatility.

The trend of the square of the worst-case density generator is also depicted in Figure 3. An increase in volatility amplifies the worst-case density generator, enabling shareholders to elevate entropy costs as the value of the investment option rises with the volatility of the cash flow shock. The influence of the growth option size (Λ_k) on the worst-case density generator is contingent on the degree of ambiguity aversion. Specifically, weak ambiguity aversion leads to a decrease in the worst-case density generator with an increase in the cash flow scaling factor Λ_k. In contrast, strong ambiguity aversion results in an increase in the worst-case density generator with an elevated Λ_k. This divergence is attributed to shareholders' pursuit of robustness: lower ambiguity aversion prompts a focus on the value of investment options, favouring a smaller density generator to mitigate investment distortion. Conversely, higher ambiguity aversion shifts attention to the value of entropy costs, favouring a larger density generator to augment entropy costs. In summary, the following proposition can be inferred.

Proposition 3. *When firms exhibit ambiguity aversion ($0 < \pi < \infty$), the expectations for type k firms, $k = h, l$, are as follows:*

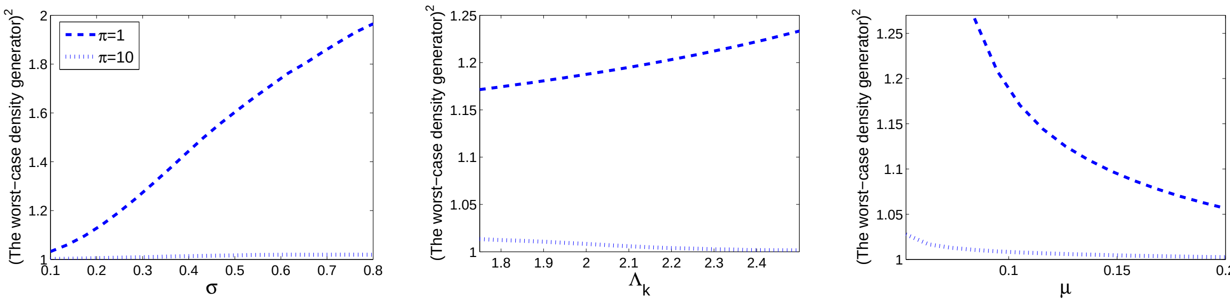
1. The worst-case density generator surpasses 1: $(\theta_k)^2 > 1$, with $\theta_k > 0$.

2. The worst-case density generator experiences an increase with the volatility of the cash flow shock σ ($\frac{\partial \theta_k}{\partial \sigma} > 0$) but undergoes a decrease with the growth rate μ ($\frac{\partial \theta_k}{\partial \mu} < 0$).

3. The monotonicity of the worst-case density generator concerning the cash flow scaling factor Λ_k is contingent on the degree of ambiguity aversion. Specifically, $\frac{\partial \theta_k}{\partial \Lambda_k} < 0$ for low ambiguity aversion (high π) and $\frac{\partial \theta_k}{\partial \Lambda_k} > 0$ for high ambiguity aversion (low π).

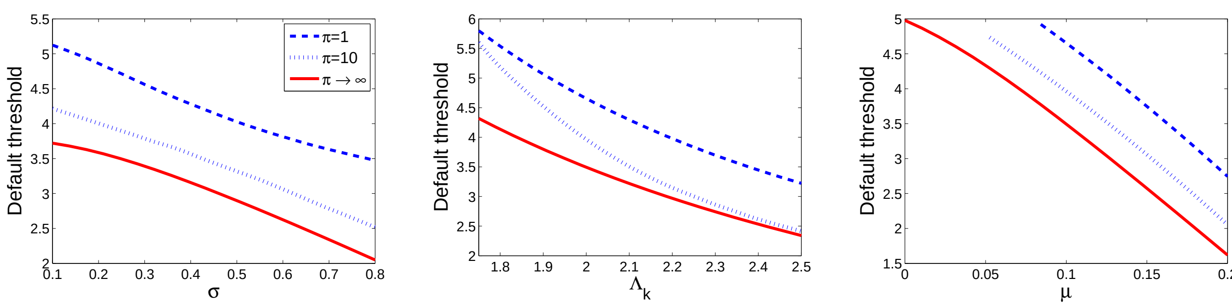
Optimal default threshold The optimal default threshold is determined by conditions 1 and 2, stating that equity is worthless in default and the equity value is maximized by setting x_k^d. Including the abandonment option in the investment and financing problem is crucial, as it effectively models the situation where shareholders are protected by limited liability, contributing significantly to the firm's financing constraints. Figure 4 illustrates that the presence of ambiguity aversion elevates the firm's default threshold and reduces the duration of the investment project. The default threshold grows with increasing ambiguity aversion, exhibiting a similar impact to that of traditional risk. This leads to the following proposition.

Proposition 4. *For type k firms, $k = h, l$, the default threshold is declining in the volatility of the cash flow shock σ, the cash flow scaling factor Λ_k, and growth rate μ ($\partial x_k^d / \partial \sigma < 0$, $\partial x_k^d / \partial \Lambda_k < 0$, and $\partial x_k^d / \partial \mu < 0$, respectively), and increasing(decreasing) in ambiguity aversion(π) $\partial x_k^d / \partial \pi < 0$.*



(a) Volatility (b) Cash flow scaling (c) Growth rate

Figure 3: The worst-case density generator. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on the square of the worst-case density generator under the first-best equilibrium scenario (symmetric information).



(a) Volatility (b) Cash flow scaling (c) Growth rate

Figure 4: Default threshold of the first-best the equilibrium. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on the default threshold under the first-best equilibrium scenario (symmetric information). The solid red line depicts the default threshold of non-ambiguous firms ($\pi \to \infty$), the blue dotted line represents that of a firm with a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line showcases that of a firm with a high degree of ambiguity aversion ($\pi = 1$).

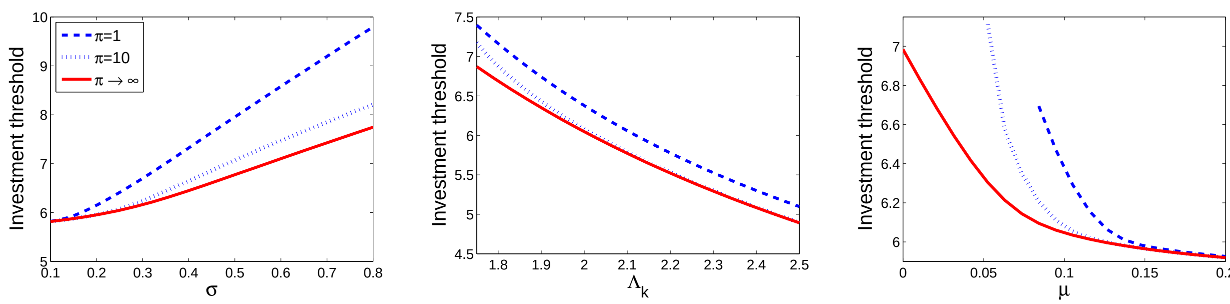
Optimal investment threshold Figure 5 displays the optimal investment threshold x_k^i. It is evident that the investment threshold under ambiguity aversion is generally higher compared to the non-ambiguity aversion scenario. The rationale is as follows: given the irreversibility of the project, there exists a waiting option value associated with investment, and the firm opts to invest only when the value of the investment option surpasses that of the waiting option. Ambiguity aversion effectively amplifies project risk, leading to an increased waiting option value and consequently causing firms to defer their investment decisions. However, Figure 5(b) demonstrates that as the firm's quality improves, the impact of ambiguity aversion on the firm's investment timing diminishes, and this effect decreases with decreasing levels of ambiguity aversion. The following proposition succinctly characterizes the optimal investment threshold.

Proposition 5. *For type k firms, $k = h, l$,*

The investment threshold increases with the volatility of the cash flow shock σ and ambiguity aversion: $\partial x_k^i / \partial \sigma > 0$ and $\partial x_k^i / \partial \pi > 0$, but decreases with the growth rate μ: $\partial x_k^i / \partial \mu < 0$.

A higher cash flow scaling factor Λ_k leads to a lower investment threshold: $\partial x_k^i / \partial \Lambda_k < 0$, particularly for low ambiguity aversion firms (high π), i.e., $\frac{\partial x_k^i}{\partial \Lambda_k \partial \pi} < 0$.

There exists a non-participating investment region induced by ambiguity aversion, where the investment threshold does not exist.



(a) Volatility

(b) Cash flow scaling

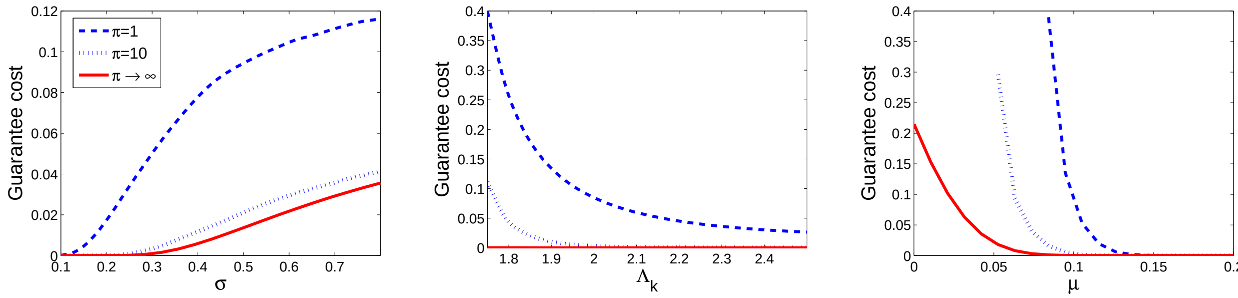
(c) Growth rate

Figure 5: Investment threshold of the first-best equilibrium. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on the investment threshold under the first-best equilibrium scenario (symmetric information). The solid red line depicts the default threshold of non-ambiguous firms ($\pi \to \infty$), the blue dotted line represents that of a firm with a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line showcases that of a firm with a high degree of ambiguity aversion ($\pi = 1$)

Guarantee cost To ensure the contractual fairness and the insurer's active involvement, the shareholder allocates a specific portion of equity to the insurer. This implies a dilution of the shareholder's equity value, with the extent of dilution worsening as the guarantee cost (β_k) intensifies. Figure 6 illustrates how the guarantee cost varies with the volatility of the cash flow shock (σ), the firms' growth potential (Λ_k), growth rate (μ), and the degree of ambiguity aversion (π). Evidently, ambiguity aversion amplifies the guarantee cost, given the corresponding decrease in equity value, as depicted in Figure 2.

Furthermore, the guarantee cost ascends with heightened volatility and descends with an increase in the growth potential and growth rate of the high-type firm. Elevated volatility diminishes the value of debts, causing an augmentation in the insurer's compensatory payment. Although equity value sees an uptick with volatility, the increment in the insurer's compensatory payment surpasses the rise in shareholder value, leading to a higher guarantee cost. Conversely, an increase in both Λ_k and μ diminishes the value of the insurer's compensatory payment, exerting a downward impact on the guarantee cost.

Proposition 6. *For type k firms, $k = h, l$, the guarantee cost increases with the volatility of the cash flow shock σ and ambiguity aversion ($\partial\beta_k/\partial\sigma > 0$ and $\partial\beta_k/\partial\pi > 0$, respectively), but decreases with the cash flow scaling factor Λ_k and growth rate μ ($\partial\beta_k/\partial\Lambda_k < 0$ and $\partial\beta_k/\partial\mu < 0$, respectively).*



(a) Volatility (b) Cash flow scaling (c) Growth rate

Figure 6: Guarantee cost of the first-best equilibrium. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on the guarantee cost under the first-best equilibrium scenario (symmetric information). The solid red line depicts the guarantee cost of non-ambiguous firms ($\pi \to \infty$), the blue dotted line represents that of a firm with a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line showcases that of a firm with a high degree of ambiguity aversion ($\pi = 1$)

4. Optimal investment strategies under asymmetric information

Subsequently, we introduce the assumption of an incomplete financial market characterized by asymmetric information and examine the repercussions of ambiguity on the firm's investment decisions. Particularly, we assume the existence of two types of firms with varying qualities in the market. As illustrated in Figure 2(b) and Figure 6(b), the equity value of high-type firms surpasses that of low-type firms, while the guarantee cost is lower than that of low-type firms.

In the context of information asymmetry, external stakeholders such as investors and insurers lack access to the true information about the firm. They cannot ascertain the firm's quality prior to investment and must rely on a belief p regarding the firm's quality. This creates an opportunity for low-type firms to mitigate guarantee costs and elevate equity values by emulating the behaviour of high-type firms. The mimicry of low-type firms hinders outsiders from updating their belief p through firm actions, imposing costs on high-type firms.

Drawing on signalling theory, we derive a perfect Bayesian equilibrium and scrutinize the investment decisions of high-type firms under the least-cost equilibrium. This analysis sheds light on the intricate dynamics at play when ambiguity interacts with information asymmetry in shaping the strategic decisions of firms.

4.1. Separating equilibrium

It is known that in the case of information asymmetry, the value of high-type firms will be damaged caused by the imitation behaviour of low-type firms. As a result, high-type have an incentive to adopt strategies to separate from low-type. Morellec and Schürhoff (2011) shows that asymmetric information lead to the advance of investment, which is contrary to the impact of ambiguity aversion on the investment, and it is expected that the cross-effect of the two will produce new results. As in Morellec and Schürhoff (2011), the objective of this section is to show that there exists a timing of investment for high-type firms that makes it possible to sustain a separating equilibrium in which the two types of firms choose different investment decisions and sign the EGS contract with honesty and trustworthiness. Under this investment threshold, the mimic cost of low-type firm is higher than the mimic income, and the market achieves a separating equilibrium. Outsiders can update their beliefs through investment timing and distinguish the two types of firms.

To determine whether there exists a timing of investment leading to a separating equilibrium, we need to check the incentive compatibility constraints (ICC). Suppose that the high-type invests at threshold x_t. If the low-type mimics the investment behaviour of the high-type, the value of the shareholders' claim in the low-type after investment is given by $(1-\beta_h(x_t))E_l^+(x_t)$. Since by mimicking, the bad type only needs to offer a proportion of shares to insurers equal to $\beta_h(x_t)$ (given by (6)) to finance the capital expenditure. This shows that mimicking the high-type reduces the cost (equity dilution) of investment. The low-type firm prefers mimicking the high-type at $x_t < x_l^i$ if:

$$(1-\beta_h(x_t))E_l^+(x_t) \geq (1-\beta_l(x_l^i))E_l^+(x_l^i)e^{-\xi_1\left(x_t - x_l^i\right)} \tag{7}$$

It is indifferent for the low-type firm between mimicking the high-type firm and waiting to invest at its own first-best timing when investing at the threshold $x\prime$ for which (7) is binding. At the same time, for $x_t < x\prime$, the low-type firm prefers to waiting until its first-best threshold than mimicking the high-type firm. In order to check whether investing below $x\prime$ is an equilibrium strategy, we need to verify an incentive-compatible condition for the good type. The incentive compatibility constraints condition for the high-type to separate from the low-type at $x_t < x_l^i$ is

$$(1-\beta_h(x_t))E_h^+(x) \geq (1-\beta_l(x_l^i))E_h^+(x_l^i)e^{-\xi_1\left(x_t - x_l^i\right)} \tag{8}$$

The lower bound $x\prime\prime$ of a separation interval solved by (8) implies that when the value of the cash flow shock is above it, the high-type firm prefers to separating from the low-type firm. That is, there exists a separating equilibrium, if the value of cash flow shock x satisfies $x\prime\prime \leq x \leq x\prime$. Identically, the unique least-cost separating equilibrium x_{sep}^i with EGS is at the lower of the thresholds $x\prime(\geq x\prime\prime)$ and x_h^i. We summarize our main results in the following proposition.

Proposition 7. (***Separating equilibrium***) *There exists a separating equilibrium in which high-type firms invest for a level of $x_t \in [x'', x']$. In the unique least-cost separating equilibrium, high-type firms invest at $x_{sep}^i = min(x', x_h^i)$ and the guarantee cost is $\beta_h(x_{sep}^i)$,* *while low-type firms invest at their first-best investment threshold x_l^i and the guarantee cost is $\beta_l(x_l^i)$. The market value before investment is*

$$V_{sep}^-(x_t) = pV_{h,sep}^-(x_t) + (1-p)V_l^-(x_t) \tag{9}$$

where $V_{h,sep}^-(x_t) \leq V_h^-(x_t)$, and the costs of signalling are measured by

$$AC_{sep}(x_t) = \frac{V_h^-(x_t) - V_{h,sep}^-(x_t)}{V_h^-(x_t)} \tag{10}$$

4.2. Pooling equilibrium

As depicted in Proposition 7, high-type firms are able to signal their private information to outsider, but signalling creates a degree of distortion in investment which incurs costs. In order to achieve the goal of maximizing the interests of shareholders, Section 4.2 studies the pooling equilibrium which provides additional firms' strategy choice. Under pooling equilibrium, financial markets are not able to distinguish among firms of different types. All firms invest at the same time and the EGS agreement specifies the same proportional share of equity for both firm types. Outsiders rely on their prior belief, p, to estimate the firm type information. This prior belief of insurers on the firm type is given by $\Lambda_p = p\Lambda_h + (1-p)\Lambda_l$ since the signal fails to reveal the true type of the firm to the counterparty. Denote the investment trigger in pooling equilibrium is x_p^i, the pooled value of the firms before investment is given by

$$V_p^-(x_t) = (E_p^+(x_p^i) + D_p^+(x_p^i))e^{-\xi_1\left(x_t - x_p^i\right)} = (1-\beta_p(x_p^i))E_p^+(x_p^i)e^{-\xi_1\left(x_t - x_p^i\right)} \tag{11}$$

where $\beta_p(x_p^i) = \frac{D_{guar,k}\left(x_p^i\right)}{E_p^+(x_p^i)}$ is guarantee cost for both firm types. According to Figure 2(b), the pooled value of the firms before investment under information asymmetry lies between the value of low-type firm and the value of high-type firm, $V_h^-(x_t) > V_p^-(x_t) > V_l^-(x_t)$, since $\Lambda_h > \Lambda_p > \Lambda_l$. Similarly, ambiguity aversion lower the pooled value of the firms.

To show that a pooling equilibrium exists, we first need to verify that pooling with the good type is an optimal strategy for the bad type. Consequently, the low-type firm prefers pooling with the high-type firm if

$$(1-\beta_p(x_t))E_l^+(x_t) \geq (1-\beta_l(x_l^i))E_l^+(x_l^i)e^{-\xi_1\left(x_t - x_l^i\right)} \tag{12}$$

According to the analysis of the separation equilibrium, (12) is true in $[x_{sep}^i, x_l^i]$. Relying only on the incentive compatibility conditions of low-type firms, we face multiplicity of equilibria. In the signal game, the high-type firm as the dominant player has the priority to make decisions. The incentive compatibility constraint (12) for the low-type is a Perfect Bayesian optimal response constraint to high-type firm's dominant strategy. Therefore, we consider the incentive compatibility constraints of high-type firm:

$$(1-\beta_p(x_t))E_h^+(x_t) \geq (1-\beta_h(x_{sep}^i))E_h^+(x_{sep}^i)e^{-\xi_1\left(x_{sep}^i - x_t\right)} \tag{13}$$

This constraint ensures that the equity value of high-type firms in the pooling equilibrium is larger than its value in the separating equilibrium. In this case, the signal costs for high-type firms are so large that the separation equilibrium is no longer the optimal equilibrium.

Pooling equilibria exist if and only if there is a threshold for which conditions (12) and (13) hold. However, there will no be a single Pareto-optimal pooling equilibrium. We need to find an investment threshold x_p^i that maximizes the present value of a type k. Depending on the dominance of high-type firms, we consider the investment threshold value x_p^i satisfied

$$\sup_{x_p^i} (1 - \beta_p(x_p^i)) E_h^+(x_p^i) e^{-\xi_1 \left(x - x_p^i\right)} \tag{14}$$

It is clear that x_p^i maximizes the value of a high-quality firm under confusion, so x_p^i satisfies the condition (14). And $x_p^i < x_l^i$ since $\Lambda_h > \Lambda_p > \Lambda_l$. Thus, we obtain a single pooling equilibrium. We then have the following existence result.

Proposition 8. (***Pooling equilibrium***) *In the pooling equilibrium, both firm types offer an identical contract $(x_p^i, \beta_p(x_p^i))$, $x_h^i < x_p^i < x_l^i$, to the insurer. Such an equilibrium exists and is Pareto-dominant if and only if (12) and (13) are true at x_p^i . Thus, the market value of high-type firm in the pooling equilibrium is $V_p^-(x) = (1 - \beta_p(x_p^i)) E_p^+(x_p^i) e^{-\xi_1 \left(x - x_p^i\right)}$, the intrinsic values is given by*

$$V_{k,p}^-(x_t) = (1 - \beta_p(x_p^i)) E_k^+(x_p^i) e^{-\xi_1 \left(x_t - x_p^i\right)} \tag{15}$$

The cost of adverse selection for pooling is

$$AC_p(x_t) = \frac{V_h^-(x_t) - V_{h,p}^-(x_t)}{V_h^-(x_t)}. \tag{16}$$

4.3. Equilibrium analysis and empirical implications

In the backdrop of asymmetric information, low-type firms endeavour to mimic high types to secure additional profits, while high-type firms aim to disclose their positive information to mitigate financing costs stemming from information asymmetry. Proposition 7 establishes the presence of a separating equilibrium, wherein high types can distinguish themselves from low types by opting for advanced investments. By investing at or below the threshold indicated by Proposition 7, the disguised cost incurred by low-type firms exceeds the disguised income, resulting in the attainment of a separating equilibrium. Outsiders can refine their beliefs through investment timing, facilitating the differentiation of the two types of firms.

However, when the signal cost is excessively high, the pooling equilibrium prevails as a Pareto-dominant outcome, wherein the decision behaviours of both low and high-type firms align completely. The market, in this case, groups the two types together based on their prior probability. Amidst the availability of two investment strategies, a pertinent question emerges: What is the value-maximizing strategy for the high type? In our model, the investment choice for the superior type is intricately determined by a trade-off between equity dilution and the more pronounced underpricing of equity associated with investment distortions. Integrating these insights, we derive the following Proposition:

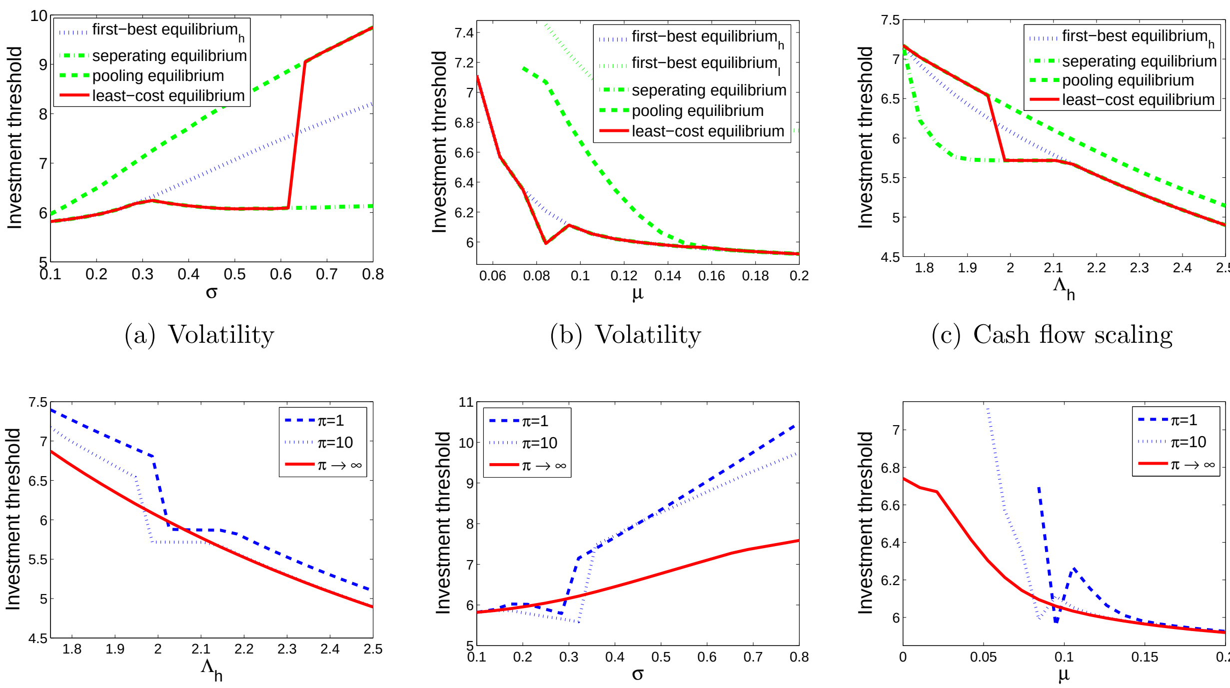
Proposition 9. ***(Least−cost equilibrium)*** *Separation is least-cost if and only if* $(1 - \beta_h(x^i_{sep}))E^+_h(x^i_{sep})e^{-\xi_1(x^i_{sep}-x^i_p)} \geq (1-\beta_p(x^i_p))E^+_h(x^i_p)$*, otherwise pooling equilibrium is least-cost.*

As previously elucidated, the presence of ambiguity aversion accentuates the divergence between the equity values of the two types of firms. This amplification implies that low-type firms stand to derive greater benefits from imitation. These considerations give rise to the following predictions associated with separating equilibrium.

Considering the firm with ambiguity aversion,

1. To attain a separating equilibrium, both the investment distortion of high-type firms and the cost of adverse selection must escalate.

2. The likelihood of a pooling equilibrium rises in the context of asymmetric information.



(a) Volatility (b) Volatility (c) Cash flow scaling

(d) Cash flow scaling (e) Growth rate (f) Growth rate

Figure 7: Investment threshold under asymmetric information. The first row (a,b,c) is Panel A, and the second row (d,e,f) is Panel B. Panel A plots investment threshold as a function of three parameters based on ambiguity aversion $\pi = 10$. Panel B plots the least-cost investment threshold under the state of no ambiguity aversion ($\pi \to \infty$), low ambiguity aversion ($\pi = 10$) and high ambiguity aversion ($\pi = 1$). The base parametrization is $r = 0.05$, $\sigma = 0.25$, $\mu = 0.1$, $F = 10/r$, $\Lambda_h = 2$, $\Lambda_l = 1.75$, $I = 20$, $\alpha = 0.5$, $\tau_i = 0.35$, $\tau_c = 0.35$, $\tau_d = 0.2$, $p = 0.5$.

To comprehensively grasp the impact of ambiguity aversion and asymmetric information on firms' investment decisions, we conduct an extensive numerical analysis, with a specific emphasis on ambiguity aversion and equilibrium distributions. Our model enables a detailed examination of the explicit effects of ambiguity aversion on firms' equilibrium investment strategies, abnormal returns following the announcement of corporate policy choices, and adverse selection costs. We delve into the investment behaviour of high-type firms as leaders to scrutinize the trade-off between different strategies.

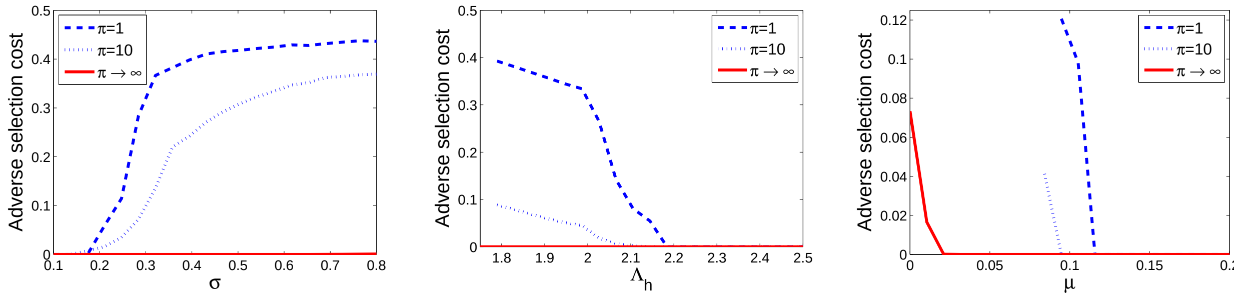
Investment threshold Continuing from the previous text, we find that this aggressive investment behaviour by low-type firms under ambiguity aversion is reflected in Panel B, supporting Prediction 1. The graph implies that ambiguity aversion prompts low-type firms to invest earlier, thereby intensifying their inclination to imitate high-types. As a consequence, high-type firms need to adopt more substantial investment distortions to achieve successful separation.

To summarise, Panel A and Panel B of Figure 7 collectively highlight that ambiguity aversion influences the investment dynamics of firms under asymmetric information. The non-monotonic effect on investment thresholds and the increased probability of pooling equilibrium being the least costly demonstrate the intricate interplay between ambiguity aversion and the strategic choices made by firms in an asymmetric information setting. These results contribute to our understanding of how ambiguity aversion shapes investment decisions and equilibrium outcomes in the presence of information asymmetry.

Cost of adverse selection The presence of asymmetric information disrupts the achievement of the first-best equilibrium for high-type firms, leading to heightened adverse selection costs and an escalation in external financing expenses. In this context, our exploration examines the determinants that influence firms' adverse selection costs. The costs of adverse selection in the separating equilibrium are precisely articulated in (10), while the corresponding costs in the pooling equilibrium are delineated in (16). In accordance with Proposition 9, the adverse selection cost in the least-cost equilibrium is defined as the minimum between AC_{sep} and AC_p. Conceptually, it becomes apparent that ambiguity aversion exacerbates adverse selection costs by amplifying investment distortions, aligning with the observations illustrated in Figure 8.

Figure 8 vividly portrays a rapid surge in adverse selection costs with increasing levels of ambiguity aversion. The blue line significantly outpaces the red line, and this discrepancy widens as volatility (σ) ascends, the growth potential of the high type (Λ_h) diminishes, and the growth rate (μ) decreases. This visual representation underscores the substantial influence of ambiguity on adverse selection costs, surpassing the impact of variables such as volatility, cash flow scaling, and growth rate. Ambiguity emerges as a pivotal factor significantly shaping the dynamics of adverse selection costs within this analytical framework.

The guarantee cost Much like the adverse selection cost, the guarantee cost under the least-cost equilibrium aligns with those under the separation and pooling equilibria. Figure 9 delineates the guarantee cost as a function of volatility, high cash flow scaling, and growth rate, with different linetypes representing varying levels of ambiguity aversion. As anticipated, the optimal guarantee cost exhibits a monotonic increase (decrease) with



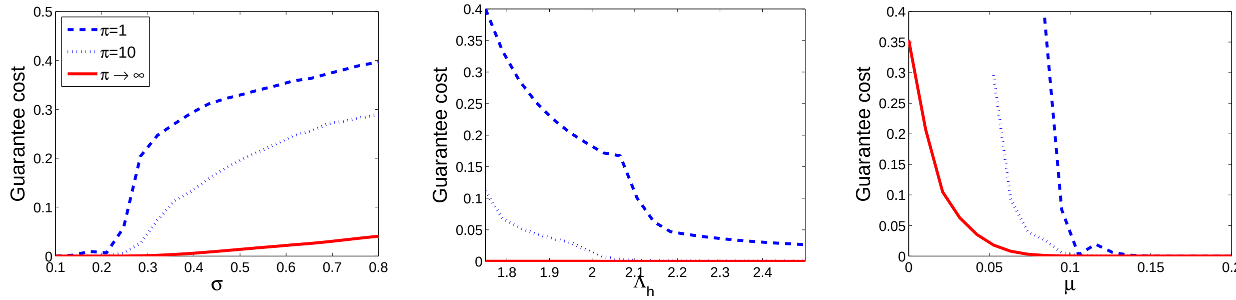
(a) Volatility (b) Cash flow scaling (c) Growth rate

Figure 8: Adverse selection cost. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on the adverse selection cost under the first-best equilibrium scenario (symmetric information). The solid red line depicts the adverse selection cost of non-ambiguous firms ($\pi \to \infty$), the blue dotted line represents that of a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line showcases that of a firm with a high degree of ambiguity aversion ($\pi = 1$)

rising ambiguity aversion (π) and the volatility of the cash flow shock. Under information asymmetry, both the separating and pooling equilibria contribute to an escalation in the guarantee cost for high-type firms. On one hand, the firm elevates the guarantee cost to convey positive information and achieve the separating equilibrium. On the other hand, the pooling equilibrium diminishes the external evaluation of the firm's quality, leading to an increase in the guarantee cost. Simultaneously, the presence of ambiguity aversion further amplifies the cost of guarantees. Comparative analysis with Figure 6 underscores that, in contrast to asymmetric information, ambiguity aversion emerges as the primary factor driving increased equity dilution for firms. Finally, as the growth potential of the high type and growth rate increase, the guarantee cost for high-type firms decreases, leading to a decline in equity dilution. This dynamic is captured by the trends observed in Figure 9.

Abnormal return In Figure 10, we adopt an alternative measure to evaluate the costs associated with asymmetric information. In the separating equilibrium, outsiders lack the ability to access firm information before investment. However, upon investment, information is revealed, enabling outsiders to leverage Bayes' rule for updating their beliefs regarding the firm's profitability. Consequently, the value of the firm undergoes changes as outsiders revise their beliefs. Let $AR_k(x_t) = (V_k^-(x_t) - V_{sep}^-(x_t))/V_{sep}^-(x_t)$ denote the jump in the value of type k, where $V_k^-(x_t)$ and $V_{sep}^-(x_t)$ are defined by Proposition 1 and Proposition 7, respectively. This change in equity value at the time of investment is termed abnormal return.

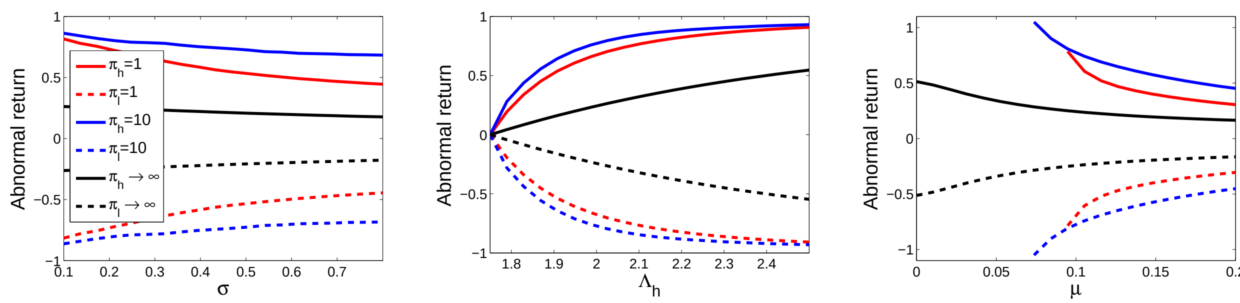
Evidently, when information is disclosed, the jump in the value of high-type firms that implement investments is positive, while the jump in value for low-type firms is negative. The solid line in Figure 10 illustrates the abnormal returns of high-type firms, and the corresponding dashed line represents the abnormal returns of low-types. It's noteworthy that the emergence of ambiguity augments the abnormal return, while an increase in ambiguity aver-



(a) Volatility (b) Cash flow scaling (c) Growth rate

Figure 9: Guarantee cost of the least-cost equilibrium. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on the guarantee cost under the first-best equilibrium scenario (symmetric information). The solid red line depicts the guarantee cost of non-ambiguous firms ($\pi \to \infty$), the blue dotted line represents that of a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line showcases that of a firm with a high degree of ambiguity aversion ($\pi = 1$)

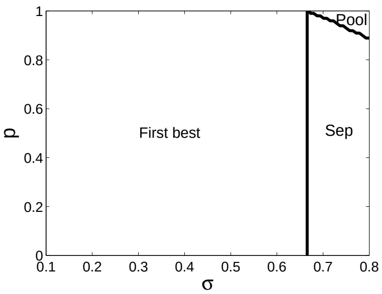
sion diminishes the abnormal return. This implies that the existence of ambiguity aversion mitigates the impact of information asymmetry on firms. However, for firms with ambiguity aversion, the impact of information asymmetry increases with the degree of ambiguity aversion. This suggests that a higher degree of ambiguity aversion results in a reduced impact of market belief changes on firms.



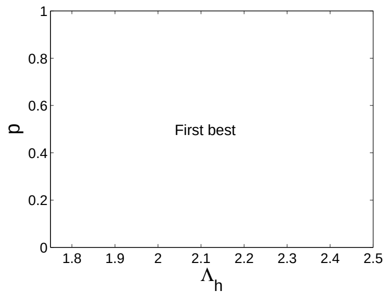
(a) Volatility (b) Cash flow scaling (c) Growth rate

Figure 10: Abnormal return. This figure illustrates the influence of project volatility (a), cash flow scaling (b), and growth rate (c) on the abnormal return under the first-best equilibrium scenario (symmetric information). The solid red line depicts the abnormal return of non-ambiguous firms ($\pi \to \infty$), the blue dotted line represents that of a low degree of ambiguity aversion ($\pi = 10$), and the blue dashed line showcases that of a firm with a high degree of ambiguity aversion ($\pi = 1$)

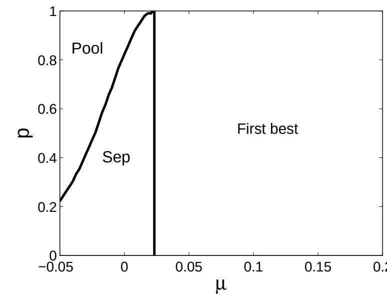
4.4. A summary of corporate equilibrium strategies and decisions



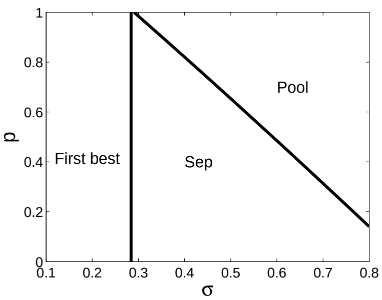
(a) Volatility



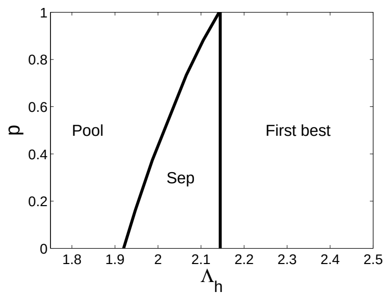
(b) Cash flow scaling



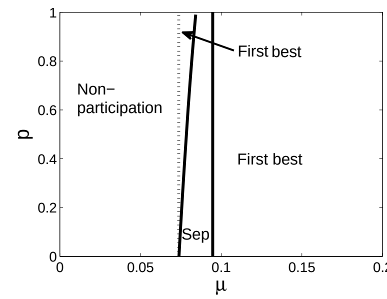
(c) Growth rate



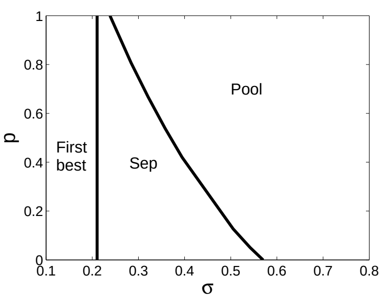
(d) Volatility



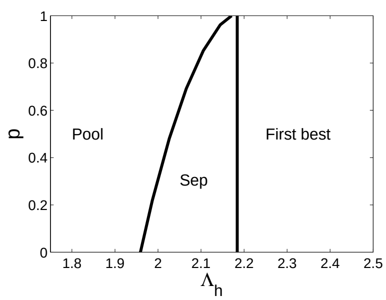
(e) Cash flow scaling



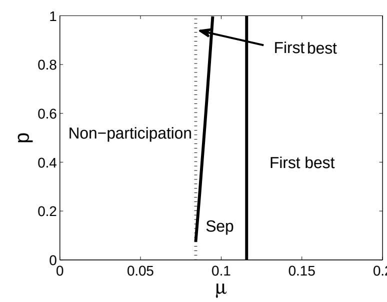
(f) Growth rate



(g) Volatility



(h) Cash flow scaling



(i) Growth rate

Figure 11: The equilibrium strategies distribution is depicted in three panels: Panel A (a, b, and c), Panel B (d, e, and f), and Panel C (g, h, and i). These panels illustrate the equilibrium strategies as a function of parameters σ, Λ_h, μ on the x-axis, and belief p on the y-axis. The panels correspond to non-ambiguous ($\pi \to \infty$), low ambiguity aversion ($\pi = 10$), and high ambiguity aversion ($\pi = 1$) firms, respectively. In the graphs, "**First best**" denotes the separating equilibrium without adverse selection cost, while "**Sep**" represents the separating equilibrium with adverse selection cost. The term "**Pooling**" refers to the pooling equilibrium, and "**Non-participation**" signifies that the firms opt not to implement the investment. The base parametrization is $r = 0.05$, $\sigma = 0.25$, $\mu = 0.1$, $F = 10/r$, $\Lambda_h = 2$, $\Lambda_l = 1.75$, $I = 20$, $\alpha = 0.5$, $\tau_i = 0.35$, $\tau_c = 0.35$, $\tau_d = 0.2$

For a deeper analysis of the ramifications of ambiguity aversion and asymmetric information, we explore the least-cost equilibrium in non-ambiguous firms, firms with low ambiguity aversion, and firms with high ambiguity aversion in markets with varying beliefs. The panels in Figure 11 delineate how ambiguity aversion and market beliefs collectively influence the least-cost equilibrium.

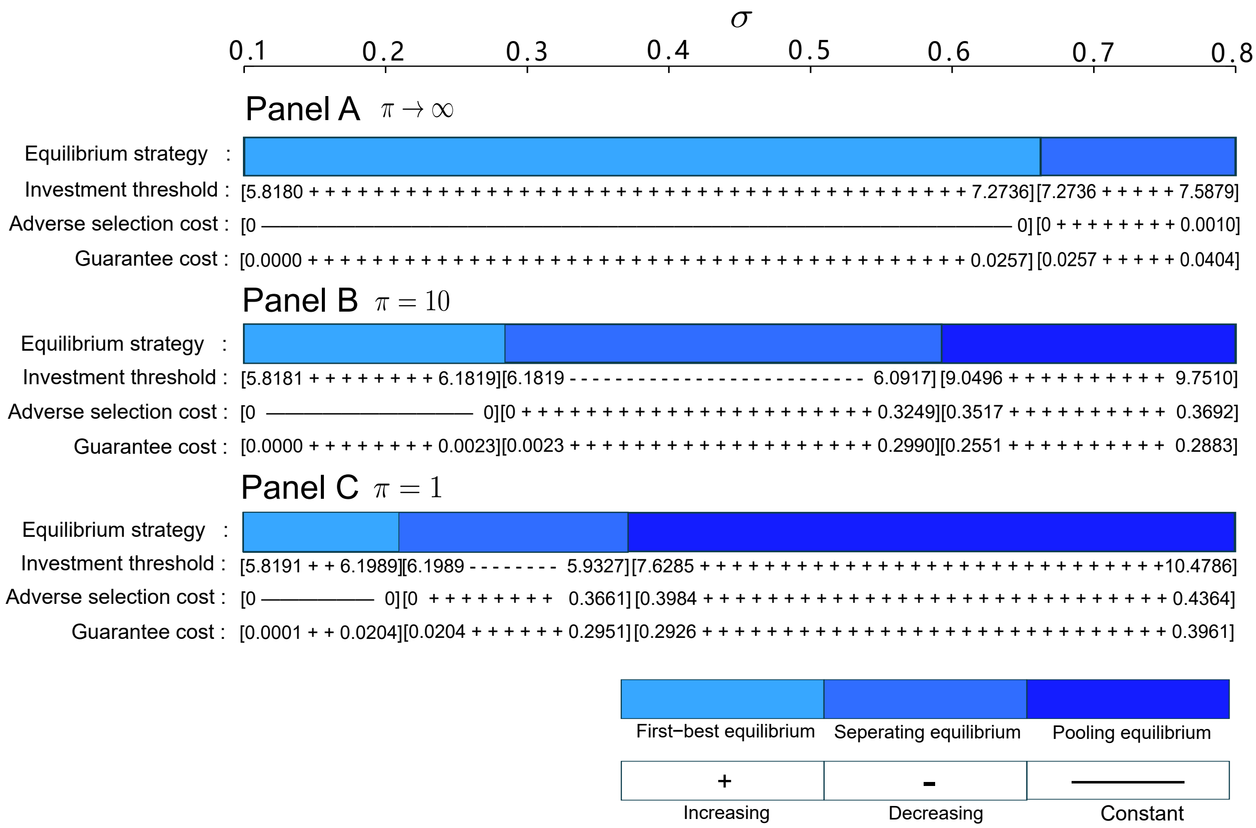


Figure 12: The impact of volatility on corporate equilibrium strategies, investment, adverse selection costs, and financing costs for three different level of ambiguity aversions (no ambiguity ($\pi \to \infty$) aversion in Panel A, the low level ($\pi = 10$) in Panel B, and the high level ($\pi = 1$) in Panel C, respectively). T light blue area represents the first-best equilibrium, the blue area shows the separating equilibrium, and the dark blue area refers to the pool equilibrium.

Figure 11 reveals that heightened ambiguity aversion can alter the equilibrium from separation to pooling, thereby expanding the region of the pooling equilibrium. This observation aligns with our earlier discussion in relation to Figure 7 Panel B. A high level of ambiguity aversion diminishes the project value, rendering firms insufficiently valued to support separation. If this effect predominates, the optimal strategy for high-type firms becomes adopting a pooling strategy. Additionally, in line with Proposition 2.2, a high level of ambiguity aversion amplifies the imitation benefits for low-type firms, prompting high-types to invest more effort in separation, which becomes overly costly. If this effect significantly prevails, the probability of a high-type firm attaining a first-best equilibrium is reduced (zero cost separation), as depicted in the panels of Figure 7 with a smaller first-best equilibrium region.

In contrast to Morellec and Schürhoff (2011) and Wang et al. (2022), Figure 11 Panel C illustrates a non-participating investment region, which expands with increasing ambiguity aversion. This phenomenon is attributed to ambiguity aversion, which diminishes project profits to the extent that costs outweigh benefits, as indicated by Proposition 2..3.

In conclusion, a comprehensive overview of the influence of volatility and ambiguity aversion on corporate equilibrium strategies, optimal investment decisions, adverse selection costs, and financing costs (specifically, guarantee costs) is succinctly presented in Figure 12. In alignment with our prior discoveries, on the one hand, heightened firm risk, or increased volatility, manifests in a less efficient separating or pooling equilibrium strategy characterized by elevated adverse selection costs, a surge in financing costs, and a pronounced tendency towards underinvestment. On the other hand, the introduction of ambiguity aversion, representing model uncertainty, exerts a notable impact by amplifying financing costs, adverse selection costs, and inducing distortion in investment decisions. This escalation in ambiguity aversion heightens the probability of inefficient separating and pooling equilibria, ultimately resulting in welfare loss. Significantly, the influence of ambiguity aversion surpasses that of firm risks, as underscored in Figure 12.

In particular, Panel A of Figure 12 delineates that firms devoid of ambiguity aversion exhibit delayed investments, with the investment threshold escalating from 5.81 to 7.59 (a 31% increase) in response to a rise in firm risk from 10% to 80%. However, when confronted with ambiguity uncertainty, the underinvestment issue intensifies significantly, with the investment threshold doubling. Furthermore, an increase in ambiguity aversion accentuates the dominance of both separating (depicted in blue) and pooling equilibrium (dark blue area) over the first-best equilibrium, resulting in an efficiency loss. This elucidation emphasizes the substantial influence of ambiguity aversion on investment dynamics and equilibrium outcomes.

5. Conclusion

When firms navigate investment decisions, the inherent uncertainty surrounding future project cash flows, commonly termed ambiguity by Ellsberg, is a critical factor. This ambiguity has been extensively scrutinized for its impact on investment decisions under conditions of perfect information in existing literature. However, the influence of ambiguity on the perfect Bayesian equilibrium in incomplete markets, particularly within the context of information asymmetry, has been somewhat overlooked.

This paper addresses this gap by constructing a dynamic multi-prior investment model where the measure of Brownian motion (shocks of firms' cash flow) is uncertain and there exists misspecification or ambiguity for the model of the firm's cash flow shock. The key finding reveals that ambiguity aversion leads to underinvestment, as the presence of ambiguity diminishes the perceived value of investment opportunities.

Notably, we demonstrate that heightened ambiguity levels incentivize low-type firms to mimic high-types, thereby expanding the pooling equilibrium. We further establish that an escalation in ambiguity intensifies guarantee costs, consequently increasing the cost of equity dilution and resulting higher financing costs.

By delineating the perfect Bayesian equilibrium of high-type firms, we ascertain that these firms can transmit a credible signal by incurring higher guarantee costs, making it more compensatory for insurers and effectively distinguishing themselves from low-type firms. Our results project that an augmentation in ambiguity levels leads to the erosion of the real option value of investment, contributing to an expansion of the pooling equilibrium where financing costs are elevated for high-quality companies. This escalation in ambiguity aversion along with increased firm risks heightens the probability of inefficient separating and pooling equilibria, ultimately resulting in welfare loss.

References

Agliardi, E., Agliardi, R., Spanjers, W., 2016. Corporate financing decisions under ambiguity: Pecking order and liquidity policy implications. Journal of Business Research 69, 6012–6020. URL: https://linkinghub.elsevier.com/retrieve/pii/S0148296316303988, doi:10.1016/j.jbusres.2016.05.016.

Anderson, E.W., Hansen, L.P., Sargent, T.J., 2003. A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection. Journal of the European Economic Association 1, 68–123. URL: https://academic.oup.com/jeea/article/2280801/A, doi:10.1162/154247603322256774.

Chen, C.C., Ho, K.C., Yan, C., Yeh, C.Y., Yu, M.T., 2023. Does ambiguity matter for corporate debt financing? Theory and evidence. Journal of Corporate Finance 80, 102425. URL: https://linkinghub.elsevier.com/retrieve/pii/S0929119923000743, doi:10.1016/j.jcorpfin.2023.102425.

Ellsberg, D., 1961. Risk, Ambiguity, and the Savage Axioms. The Quarterly Journal of Economics 75, 643–669. URL: http://www.jstor.org/stable/1884324, doi:10.2307/1884324. publisher: Oxford University Press.

Epstein, L.G., Ji, S., 2013. Ambiguous Volatility and Asset Pricing in Continuous Time. Review of Financial Studies 26, 1740–1786. URL: https://academic.oup.com/rfs/article-lookup/doi/10.1093/rfs/hht018, doi:10.1093/rfs/hht018.

Flannery, M.J., 1986. Asymmetric Information and Risky Debt Maturity Choice. The Journal of Finance 41, 19–37. URL: https://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.1986.tb04489.x, doi:10.1111/j.1540-6261.1986.tb04489.x.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. Journal of Mathematical Economics 18, 141–153. URL: https://www.sciencedirect.com/science/article/pii/0304406889900189, doi:10.1016/0304-4068(89)90018-9.

Hansen, L., Sargent, T.J., 2001. Robust Control and Model Uncertainty. American Economic Review 91, 60–66. URL: https://www.aeaweb.org/articles?id=10.1257/aer.91.2.60, doi:10.1257/aer.91.2.60.

Hansen, L.P., Sargent, T.J., Turmuhambetova, G., Williams, N., 2006. Robust control and model misspecification. Journal of Economic Theory 128, 45–90. URL: https://www.sciencedirect.com/science/article/pii/S0022053105001973, doi:https://doi.org/10.1016/j.jet.2004.12.006.

Knight, F.H., 1921. Risk, Uncertainty and Profit. Houghton Mifflin Co, Boston, MA. URL: http://www.econlib.org/library/Knight/knRUP.html.

Ling, A., Miao, J., Wang, N., 2021. Robust Financial Contracting and Investment. Working Paper URL: http://www.nber.org/papers/w28367, doi:10.3386/w28367.

Luo, P., Tian, Y., 2022. Investment, payout, and cash management under risk and ambiguity. Journal of Banking & Finance 141, 106551. URL: https://linkinghub.elsevier.com/retrieve/pii/S0378426622001479, doi:10.1016/j.jbankfin.2022.106551.

Maccheroni, F., Marinacci, M., Rustichini, A., 2006. Ambiguity Aversion, Robustness, and the Variational Representation of Preferences. Econometrica 74, 1447–1498. URL: http://doi.wiley.com/10.1111/j.1468-0262.2006.00716.x, doi:10.1111/j.1468-0262.2006.00716.x.

Morellec, E., Schürhoff, N., 2011. Corporate investment and financing under asymmetric information. Journal of Financial Economics 99, 262–288. URL: https://linkinghub.elsevier.com/retrieve/pii/S0304405X10002096, doi:10.1016/j.jfineco.2010.09.003.

Myers, S.C., Majluf, N.S., 1984. Corporate financing and investment decisions when firms have information that investors do not have. Journal of Financial Economics 13, 187–221. URL: https://www.sciencedirect.com/science/article/pii/0304405X84900230, doi:10.1016/0304-405x(84)90023-0.

Neamtiu, M., Shroff, N., White, H.D., Williams, C.D., 2014. The Impact of Ambiguity on Managerial Investment and Cash Holdings. Journal of Business Finance & Accounting 41, 1071–1099. URL: https://onlinelibrary.wiley.com/doi/10.1111/jbfa.12079, doi:10.1111/jbfa.12079.

Nishimura, K.G., Ozaki, H., 2007. Irreversible investment and Knightian uncertainty. Journal of Economic Theory 136, 668–694. URL: https://linkinghub.elsevier.com/retrieve/pii/S0022053107000385, doi:10.1016/j.jet.2006.10.011.

Song, P., Zhang, H., Zhao, Q., 2021. Innovative Credit Guarantee Schemes with equity-for-guarantee swaps. International Review of Financial Analysis 77, 101809. URL:

https://linkinghub.elsevier.com/retrieve/pii/S1057521921001435, doi:10.1016/j.irfa.2021.101809.

Thijssen, J.J., 2011. Incomplete markets, ambiguity, and irreversible investment. Journal of Economic Dynamics and Control 35, 909–921. URL: https://linkinghub.elsevier.com/retrieve/pii/S0165188910002575, doi:10.1016/j.jedc.2010.12.001.

Wang, H., Yang, Z., Zhang, H., 2015. Entrepreneurial finance with equity-for-guarantee swap and idiosyncratic risk. European Journal of Operational Research 241, 863–871. URL: https://linkinghub.elsevier.com/retrieve/pii/S0377221714007346, doi:10.1016/j.ejor.2014.09.013.

Wang, Y., Zhang, H., Zhao, Z., 2022. SME investment and financing under asymmetric information. European Financial Management 28, 1347–1375. URL: https://onlinelibrary.wiley.com/doi/10.1111/eufm.12342, doi:10.1111/eufm.12342.

Yang, Z., Zhang, H., 2013. Optimal capital structure with an equity-for-guarantee swap. Economics Letters 118, 355–359. URL: https://linkinghub.elsevier.com/retrieve/pii/S0165176512006131, doi:10.1016/j.econlet.2012.11.023.